# **Real-Time Signal Processing and State Estimation for Spaceflight Applications**

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## Abstract

This thesis presents real-time data processing in spaceflight sensing systems under onboard computational constraints. Field-Programmable Gate Array (FPGA) based architectures are leveraged for high-throughput, low-latency operations while addressing inherent performance degradation due to finite-precision arithmetic.

The first part of this thesis develops a signal processing front-end for interferometric optomechanical sensing. A digital phase measurement system is conceived to enable high-precision phase **readout and tracking** with minimal noise floor. Simulations are presented to analyze system performance while experimental validation demonstrates the precision and reliability of the phase sensing system.

The second part focuses on **optimal state estimation back-end for inertial navigation**. Kalman filter algorithms are reformulated to incorporate finite-precision numerical errors in states, inputs, and measurements. Performance trade-offs with numerical precision are captured to provide insights into the best possible filter accuracy achievable for a given numerical representation. Numerical simulations and experimental results underscore the significance of modeling quantization errors into state estimation pipelines for embedded implementations.



Figure 1. Interferometric sensing systems require high-frequency data processing operations for science readout. Field Programmable Gate Arrays (FPGAs) enable the demanding Digital Signal Processing (DSP) and navigation (Nav) algorithms to be deployed at the edge.

# Motivation Noise ADC $\tilde{y} = y + v$ y = h(x)Measurement

Figure 2. Quantization errors in measurements (states, and inputs) degrade signal-to-noise ratio (SNR) in fixed-point sensing and navigation systems.

**Challenge:** High-precision optical sensors require low-cost, resource-efficient DSP implementations, but quantization noise from fixed-point arithmetic degrades SNR and navigation accuracy. **Research Questions:** 

- 1. Can low-cost fixed-point DSP systems achieve **high-precision optical sensor requirements**?
- 2. How can quantization-aware algorithms **optimize SNR** in resource-constrained systems?
- 3. Can navigation filter performance be enhanced by **modeling finite-precision hardware errors**?

Goal: Develop signal processing systems that reduce noise floors while ensuring stable, costeffective DSP operation.

### Key Contributions

This research advances signal processing methods for precision spaceflight applications through integrated system and algorithm design:

- High-Precision Optical Sensing: Real-time high-rate DSP system achieving microradians phase measurement precision with fixed-point arithmetic.
- Quantized Navigation Algorithms: Novel Kalman filter variants (QDKF, QSRKF) that explicitly model and compensate for finite-precision errors in states, inputs, and measurements.

**Impact**: Enables low-cost, high-precision sensing and navigation systems for future space missions requiring both computational efficiency and scientific accuracy.



```
z = \mathbb{Q}[\tilde{y}]
= y + v + \epsilon_q
= y + \epsilon
```

- Phase Measurement System
- Intersatellite laser interferometry detects gravitational accelerations by measuring spacecraft motion through ultra-precise optical phase metrology.
- Gravitational forces  $\rightarrow$  test mass displacement  $\rightarrow$  Phase change in beat note.
- Laser Interferometric Space Antenna (LISA) requirements: Phase measurement precision of  $6\,\mu$ rad/ $\sqrt{\text{Hz}}$  enabling displacement sensitivity of  $\approx 10^{-12}$ m/ $\sqrt{\text{Hz}}$ .

### Phasemeter System Design

The digital phasemeter system performs real-time phase measurements using an FPGA Systemon-Chip (SoC) platform (Fig. 3). High-rate DSP operations execute on the FPGA fabric at the ADC sampling frequency. A multi-stage decimation filter chain, implemented the programmable logic (PL) and the processing system (PS) reduces the data rate to 3.81 Hz for precision science readout.



Figure 3. FPGA-SoC for optical phase metrology: The DSP processor (Fig. 4) on the FPGA fabric computes differential phase between input and reference channels for readout.

The FPGA implements dual instances of all-digital phase-locked loop (ADPLL) cores to compute the phase difference between the input and reference channels (Fig. 4). An ADPLL is a closedloop feedback control system that locks onto the frequency of an incoming signal and provides instantaneous phase values of the input.



Figure 4. Phase measurement principle: Independent ADPLL cores track input and reference signals. The differential phase measurement  $\Delta \varphi = \phi_1 - \phi_2$  provides the phase readout.

# Experiments

The phasemeter hardware is verified using Simulink<sup>®</sup> floating-point simulations. Results from the RF testbench demonstrate that, within the measurement band (0.1 mHz-1 Hz), the phasemeter meets the LISA precision requirements above 1 mHz. NSF: Noise Shaping Function.



Figure 5. RF Benchtop testing setup for phasemeter hardware demonstration.

 $\left[ \sum_{i=1}^{\infty} 10^{-1} \right]$  $10^{-1}$ 





Figure 6. Phase noise performance compared  $6\,\mu$ rad/ $\sqrt{\text{Hz}}$  LISA requirement.





### **Sensor Dynamics and State Estimation**

• 1 DoF accelerometer sensor dynamics: perturbed harmonic oscillator

$$\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = g(\dot{b}(t) = n_u(t)$$

 $b(t) + b(t) + n_v(t)$ ( Mass displacement x for acceleration g) (► Stochastic bias: Wiener process) • Discretized dynamics with quantized states, inputs, and measurements ( $\mathbb{Q}[\cdot]$ ).

$$\begin{aligned} \mathbf{X}_{k+1} &= \mathbf{\Phi}(t_{k+1}, t_k) \\ y_k &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbb{Q}[\mathbf{X}_k] \\ \mathbb{Q}[y_k] &= y_k + \epsilon_{y,k} \end{aligned}$$

 Quantized state estimation: ► Kalman gain augments round-off error covariances as optimal weighting factors ► Amplifies covariance updates accommodating additional uncertainties due to finite-precision realization  $1-1\mathbf{T}\mathbf{T}\mathbf{D}-1$  ( $\tilde{\mathbf{r}}$   $\tilde{\mathbf{i}}$ )

$$\hat{\tilde{\mathbf{X}}}(k) = [\mathbf{H}_k^T \mathbf{P}_{\mu\mu}^{-1} \mathbf{H}_k]$$
$$\mathbf{P}_{\mu\mu} = \mathbb{E}[\mu\mu^T] = \mathbf{H}$$

# Simulations and Hardware Results

- $\blacktriangleright$  Embedding quantization noise into filters  $\rightarrow$  Reduced errors & improved confidence (Fig. 7). Steady-state covariance analysis: measurement precision v. model uncertainty and estimation errors (Fig. 8)  $\rightarrow$  Important tool for sensor selection, parameter modeling, and tuning.



Figure 7. Estimation errors and  $3\sigma$  bounds from DKF and QDKF with 12-bit measurements.





Figure 10. FPGA v. double-precision simulations. Figure 9. Nav filter operations on FPGA-SoC.

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### **Quantized State Estimation**

- Optimal quantized filtering methods for finite-precision in states, inputs, and measurements.
- Application: Estimation of forcing input (acceleration) from optical interferometry.

- $\mathbf{Y}_k \mathbb{Q}[\mathbf{X}_k] + \mathbf{\Gamma}(t_{k+1}, t_k) \mathbb{Q}[g_k] + \mathbf{w}_k$  $[\mathbf{X}_k] + \nu_k$
- (► Quantized states) (► Discrete measurements) (  $\land$  A/D conversion error:  $\epsilon_{u,k}$ )

J ⁺H	$\mathbf{L}_{k} \mathbf{P}_{\mu\mu}$	$\boldsymbol{\mu}(\mathbf{y}-\boldsymbol{\eta}_k)$	(b)	
$\mathbf{\Sigma}_{k} \mathbf{\Sigma}_{ ilde{\mathbf{X}}}$	$\mathbf{H}_{k}^{T}$ +	$\boldsymbol{\eta}_k \Sigma_{\hat{b}} \boldsymbol{\eta}_k^T$	$\mathbf{P} + \mathbf{P}_{\tilde{\boldsymbol{\nu}}\tilde{\boldsymbol{\nu}}}$	$+\Sigma$

# (► Minimum variance state estimate) (► Error covariance)





- Figure 8. Steady-state: (Left)  $1\sigma$  contours. (Right) Mahalanobis distance of estimates.
- $\blacktriangleright$  Hardware implementation and testing  $\rightarrow$  benchmarks estimation accuracy for flight compute.



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### References

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